JANUARY 2007

MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

03 JANUARY 2007 (a.m.)

INSTRUCTIONS TO CANDIDATES

1. Answer ALL questions in Section I, and ANY TWO in Section II.
2. Write your answers in the booklet provided.
3. All working must be shown clearly.
4. A list of formulae is provided on page 2 of this booklet.

Examination Materials

Electronic calculator (non-programmable)
Geometry set
Mathematical tables (provided)
Graph paper (provided)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

LIST OF FORMULAE

Volume of a prism: $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of cylinder: $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

Volume of a right pyramid: $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference: $C = 2\pi r$ where r is the radius of the circle.

Area of a circle: $A = \pi r^2$ where r is the radius of the circle.

Area of trapezium: $A = \frac{1}{2}(a + b)h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

Roots of quadratic equations: If $ax^2 + bx + c = 0$,

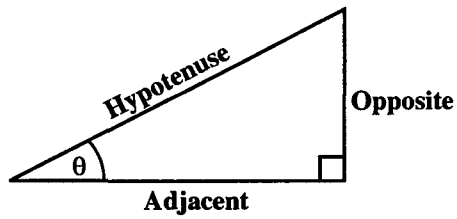
then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

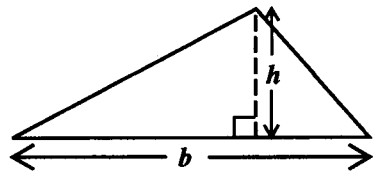
$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



Area of triangle: Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height

Area of $\Delta ABC = \frac{1}{2}ab \sin C$

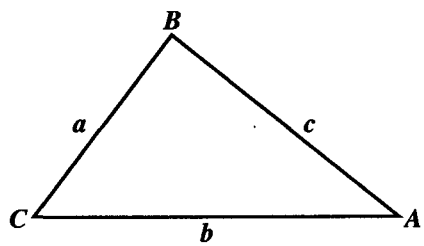


Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a + b + c}{2}$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$



SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, evaluate

(i) $5.24\,(4 - 1.67)$ **(2 marks)**

(ii) $\dfrac{1.68}{1.5^2 - 1.45}$ **(3 marks)**

(b) A sum of money is shared between Aaron and Betty in the ratio 2 : 5. Aaron received \$60. How much money was shared **altogether**? **(3 marks)**

(c) In St. Vincent, 3 litres of gasoline cost EC\$10.40.

(i) Calculate the cost of 5 litres of gasoline in St. Vincent, **stating your answer correct to the nearest cent.** **(2 marks)**

(ii) How many litres of gasoline can be bought for EC \$50.00 in St. Vincent?

Give your answer correct to the nearest whole number. **(2 marks)**

Total 12 marks

2. (a) If $a = 2$, $b = -3$ and $c = 4$, evaluate

(i) $ab - bc$ **(1 mark)**

(ii) $b(a - c)^2$ **(2 marks)**

(b) Solve for x where $x \in \mathbf{Z}$:

(i) $\frac{x}{2} + \frac{x}{3} = 5$ **(3 marks)**

(ii) $4 - x \leq 13$ **(3 marks)**

(c) The cost of ONE muffin is $\$m$.
The cost of THREE cupcakes is $\$2m$.

(i) Write an algebraic expression in m for the cost of:

a) FIVE muffins **(1 mark)**

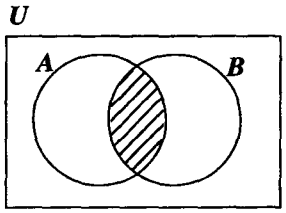
b) SIX cupcakes **(1 mark)**

(ii) Write an equation, in terms of m, to represent the following information.

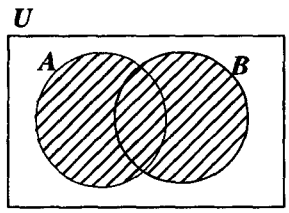
The TOTAL cost of 5 muffins and 6 cupcakes is \$31.50. **(1 mark)**

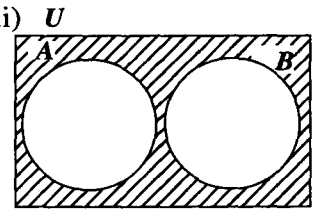
Total 12 marks

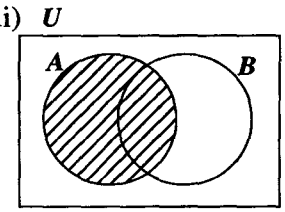
3. (a) Describe, using set notation only, the shaded regions in each Venn diagram below. **The first one is done for you.**



$A \cap B$

(i)


(ii)


(iii)


(3 marks)

(b) The following information is given.

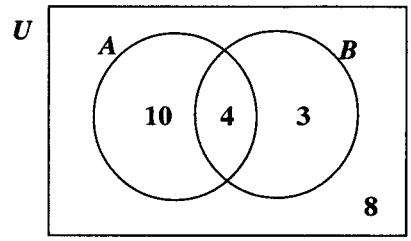
U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

P = {prime numbers}

Q = {odd numbers}

Draw a Venn diagram to represent the information above. **(3 marks)**

(c) The Venn diagram below shows the number of elements in each region.



Determine how many elements are in EACH of the following sets:

(i) $A \cup B$ **(1 mark)**

(ii) $A \cap B$ **(1 mark)**

(iii) $(A \cap B)'$ **(1 mark)**

(iv) U **(1 mark)**

Total 10 marks

4. (a) (i) Using a pencil, ruler and a pair of compasses only, construct ΔABC with $BC = 6$ cm and $AB = AC = 8$ cm. **(3 marks)**

All construction lines must be clearly shown.

(ii) Draw a line segment AD such that AD meets BC at D and is perpendicular to BC. **(2 marks)**

(iii) Measure and state

a) the length of the line segment AD **(1 mark)**

b) the size of angle ABC **(1 mark)**

(b) P is the point (2, 4) and Q is the point (6, 10).

Calculate

(i) the gradient of PQ **(2 marks)**

(ii) the midpoint of PQ. **(2 marks)**

Total 11 marks

5. An answer sheet is provided for this question.

(a) f and g are functions defined as follows

$f: x \to 7x + 4$

$g: x \to \frac{1}{2x}$

Calculate

(i) $g(3)$ **(1 mark)**

(ii) $f(-2)$ **(2 marks)**

(iii) $f^{-1}(11)$ **(2 marks)**

(b) On the answer sheet provided, ΔABC is mapped onto $\Delta A'B'C'$ under a reflection.

(i) Write down the equation of the mirror line. **(1 mark)**

$\Delta A'B'C'$ is mapped onto $\Delta A''B''C''$ by a rotation of 180° about the point (5, 4).

(ii) Determine the coordinates of the vertices of $\Delta A''B''C''$. **(3 marks)**

(iii) State the transformation that maps ΔABC onto $\Delta A''B''C''$. **(2 marks)**

Total 11 marks

6. The table below shows a frequency distribution of the scores of 100 students in an examination.

Scores	Frequency	Cumulative Frequency
21 - 25	5	5
26 - 30	18	
31 - 35	23	
36 - 40	22	
41 - 45	21	
46 - 50	11	100

(i) Copy and complete the table above to show the cumulative frequency for the distribution. **(2 marks)**

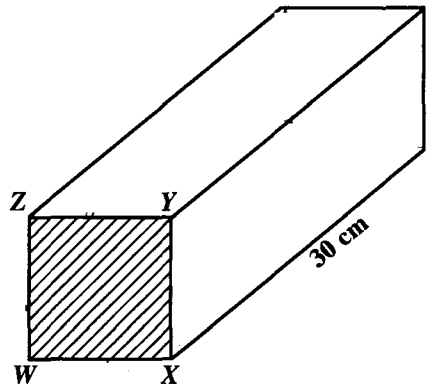
(ii) Using a scale of 2 cm to represent a score of 5 on the horizontal axis and a scale of 2 cm to represent 10 students on the vertical axis, draw a cumulative frequency curve of the scores. Start your horizontal scale at 20. **(6 marks)**

(iii) Using the cumulative frequency curve, determine the median score for the distribution. **(2 marks)**

(iv) What is the probability that a student chosen at random has a score greater than 40? **(2 marks)**

Total 12 marks

7. (a) The diagram below, **not drawn to scale**, shows a prism of length 30 cm. The cross-section *WXYZ* is a square with area 144 cm^2.

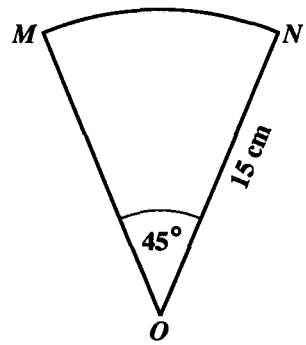


Calculate

(i) the volume, in cm^3, of the prism **(2 marks)**

(ii) the total surface area, in cm^2, of the prism. **(4 marks)**

(b) The diagram below, **not drawn to scale**, shows the sector of a circle with centre O. $\angle MON = 45°$ and $ON = 15$ cm.



Use $\pi = 3.14$

Calculate, giving your answer correct to 2 decimal places

(i) the length of the minor arc *MN* **(2 marks)**

(ii) the perimeter of the figure *MON* **(2 marks)**

(iii) the area of the figure *MON*. **(2 marks)**

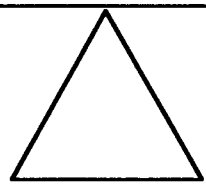
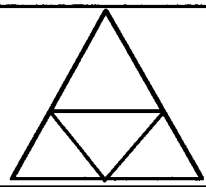
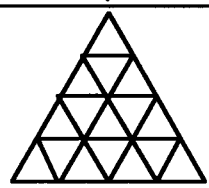
Total 12 marks

8. A large equilateral triangle is subdivided into a set of smaller equilateral triangles by the following procedure:

The midpoints of the sides of each equilateral triangle are joined to form a new set of smaller triangles.

The procedure is repeated my times.

The table below shows the results when the above procedure has been repeated twice, that is, when $n = 2$.

n	Result after each step	No. of triangles formed
0		1
1		4
2		16
3		(i)
6		(ii)
(iii)		65536
m		(iv)

(i) Calculate the number of triangles formed when $n = 3$. **(2 marks)**

(ii) Determine the number of triangles formed when $n = 6$. **(2 marks)**

A shape has 65 536 small triangles.

(iii) Calculate the value of n. **(3 marks)**

(iv) Determine the number of small triangles in a shape after carrying out the procedure m times. **(3 marks)**

Total 10 marks

SECTION II

Answer TWO questions in this section

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) Factorise completely

(i) $2p^2 - 7p + 3$ **(1 mark)**

(ii) $5p + 5q + p^2 - q^2$ **(2 marks)**

(b) Expand $(x + 3)^2 (x - 4)$, writing your answer in descending powers of x. **(3 marks)**

(c) Given $f(x) = 2x^2 + 4x - 5$

(i) write $f(x)$ in the form $f(x) = a(x + b)^2 + c$ where $a, b, c \in \mathbf{R}$ **(3 marks)**

(ii) state the equation of the axis of symmetry **(1 mark)**

(iii) state the coordinates of the minimum point **(1 mark)**

(iv) sketch the graph of $f(x)$ **(2 marks)**

(v) on the graph of $f(x)$ show clearly

a) the minimum point **(1 mark)**

b) the axis of symmetry. **(1 mark)**

Total 15 marks

10. An answer sheet is provided for this question.

Pam visits the stationery store where she intends to buy x pens and y pencils.

(a) Pam must buy at least 3 pens.

(i) Write an inequality to represent this information. **(1 mark)**

The TOTAL number of pens and pencils must NOT be more than 10.

(ii) Write an inequality to represent this information. **(2 marks)**

EACH pen costs \$5.00 and EACH pencil costs \$2.00. More information about the pens and pencils is represented by:

$$5x + 2y \leq 35$$

(iii) Write the information represented by this inequality as a sentence in your own words. **(2 marks)**

(b) (i) On the answer sheet provided, draw the graph of the TWO inequalities obtained in (a) (i) and (a) (ii) above. **(3 marks)**

(ii) Write the coordinates of the vertices of the region that satisfies the four inequalities (including $y \geq 0$). **(2 marks)**

(c) Pam sells the x pens and y pencils and makes a profit of \$1.50 on EACH pen and \$1.00 on EACH pencil.

(i) Write an expression in x and y to represent the profit Pam makes. **(1 mark)**

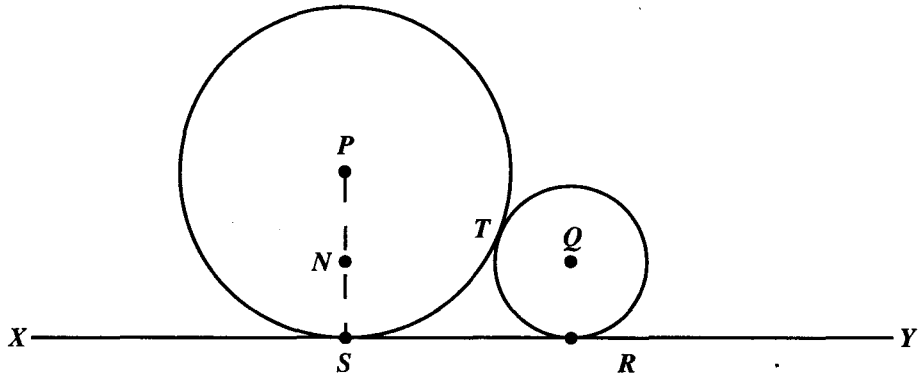
(ii) Calculate the maximum profit Pam makes. **(2 marks)**

(iii) If Pam buys 4 pens, show **on your graph** the maximum number of pencils she can buy. **(2 marks)**

Total 15 marks

GEOMETRY AND TRIGONOMETRY

11. (a) Two circles with centres P and Q and radii 5 cm and 2 cm respectively are drawn so that they touch each other at T and a straight line XY at S and R.



(i) State, with a reason,

a) why PTQ is a straight line **(2 marks)**

b) the length PQ **(2 marks)**

c) why PS is parallel to QR. **(2 marks)**

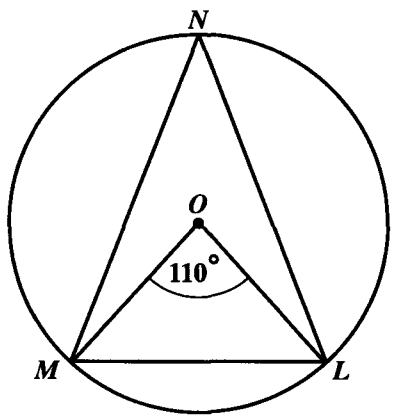
(ii) N is a point on PS such that QN is perpendicular to PS.

Calculate

a) the length PN **(2 marks)**

b) the length RS. **(2 marks)**

(b) In the diagram below, **not drawn to scale**, O is the centre of the circle. The measure of angle LOM is 110°.



Calculate, giving reasons for your answers, the size of EACH of the following angles

(i) $\angle MNL$ **(2 marks)**

(ii) $\angle LMO$ **(3 marks)**

Total 15 marks

12. A boat leaves a dock at point A and travels for a distance of 15 km to point B on a bearing of 135°.

The boat then changes course and travels for a distance of 8 km to point C on a bearing of 060°.

(a) Illustrate the above information in a clearly labelled diagram. **(2 marks)**

The diagram should show the

(i) north direction **(1 mark)**

(ii) bearings 135° and 060° **(2 marks)**

(iii) distances 8 km and 15 km. **(2 marks)**

(b) Calculate

(i) the distance AC **(3 marks)**

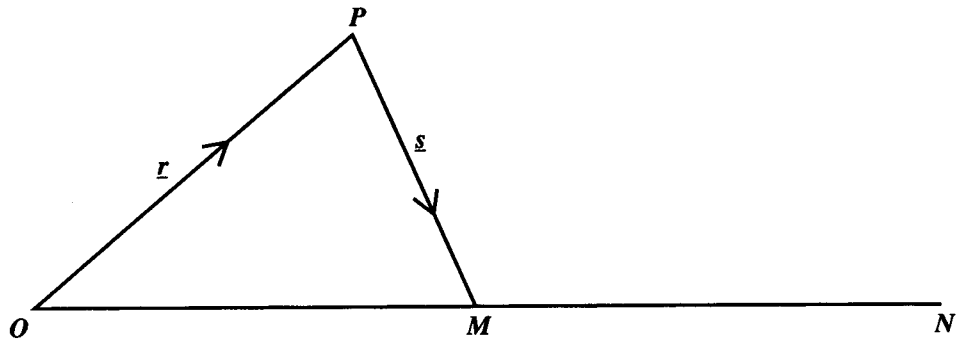
(ii) $\angle BCA$ **(3 marks)**

(iii) the bearing of A from C. **(2 marks)**

Total 15 marks

VECTORS AND MATRICES

13. In the diagram below, M is the midpoint of $\overrightarrow{ON}$.



(a) (i) Sketch the diagram above in your answer booklet and insert the point X on $\overrightarrow{OM}$ such that $\overrightarrow{OX} = \frac{1}{3}\overrightarrow{OM}$. **(1 mark)**

(ii) Produce PX to Q such that $\overrightarrow{PX} = 4\overrightarrow{XQ}$. **(1 mark)**

(b) Write the following in terms of $\underline{r}$ and $\underline{s}$.

(i) $\overrightarrow{OM}$ **(2 marks)**

(ii) $\overrightarrow{PX}$ **(3 marks)**

(iii) $\overrightarrow{QM}$ **(4 marks)**

(c) Show that $\overrightarrow{PN} = 2\overrightarrow{PM} + \overrightarrow{OP}$ **(4 marks)**

Total 15 marks

14. (a) Given that D = $\begin{bmatrix} 1 & 9p \\ p & 4 \end{bmatrix}$ is a singular matrix, determine the value(s) of p. **(4 marks)**

(b) Given the linear equations

$2x + 5y = 6$
$3x + 4y = 8$

(i) Write the equations in the form $AX = B$ where A, X and B are matrices. **(2 marks)**

(ii) a) Calculate the determinant of the matrix A. **(2 marks)**

b) Show that $A^{-1} = \begin{pmatrix} \frac{-4}{7} & \frac{5}{7} \\ \frac{3}{7} & \frac{-2}{7} \end{pmatrix}$. **(2 marks)**

c) Use the matrix A^{-1} to solve for x and y. **(5 marks)**

Total 15 marks

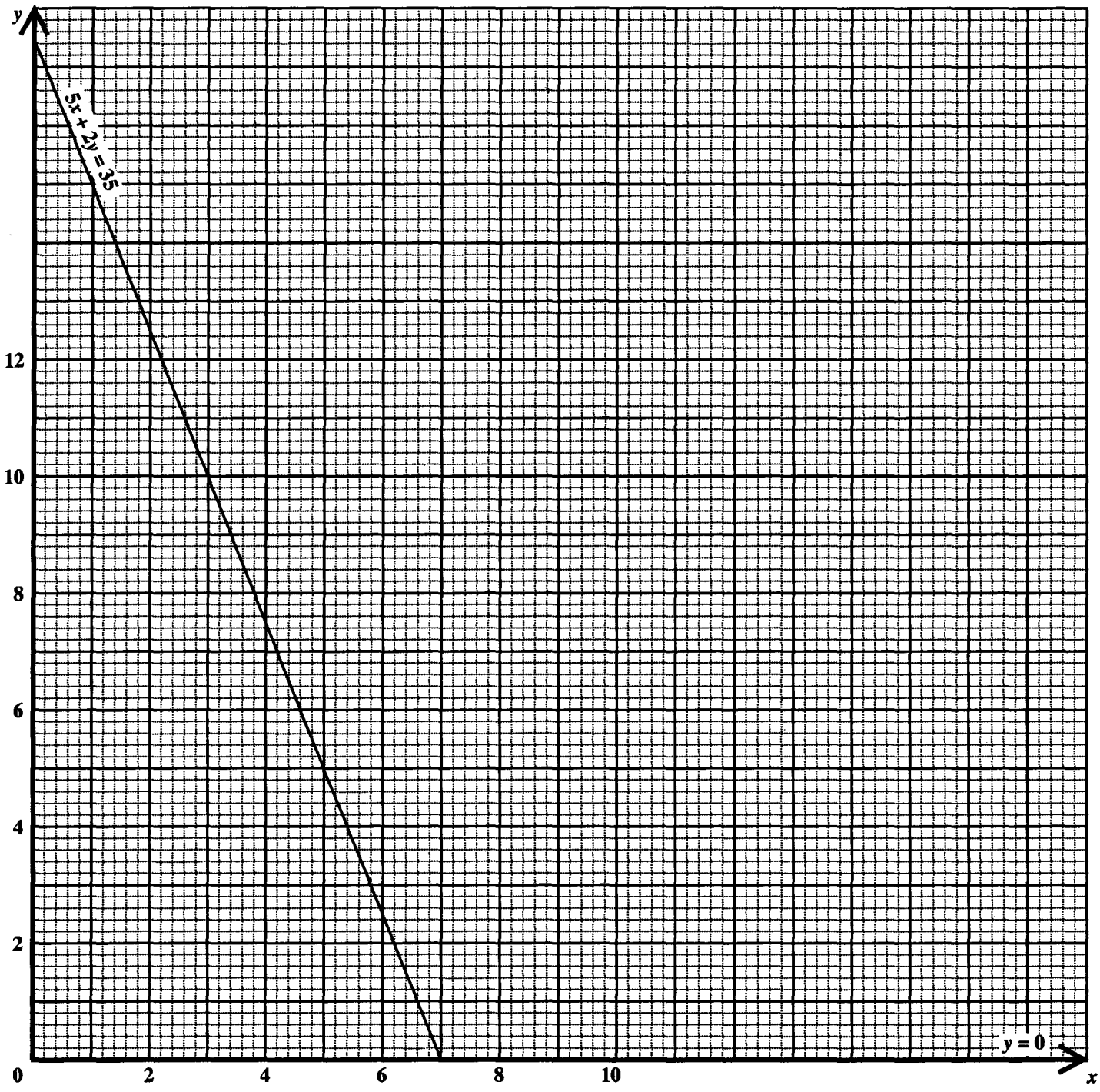
END OF TEST

JANUARY 2007

MATHEMATICS

Paper 02 – General Proficiency

Answer Sheet for Question 10 **Candidate Number .....**



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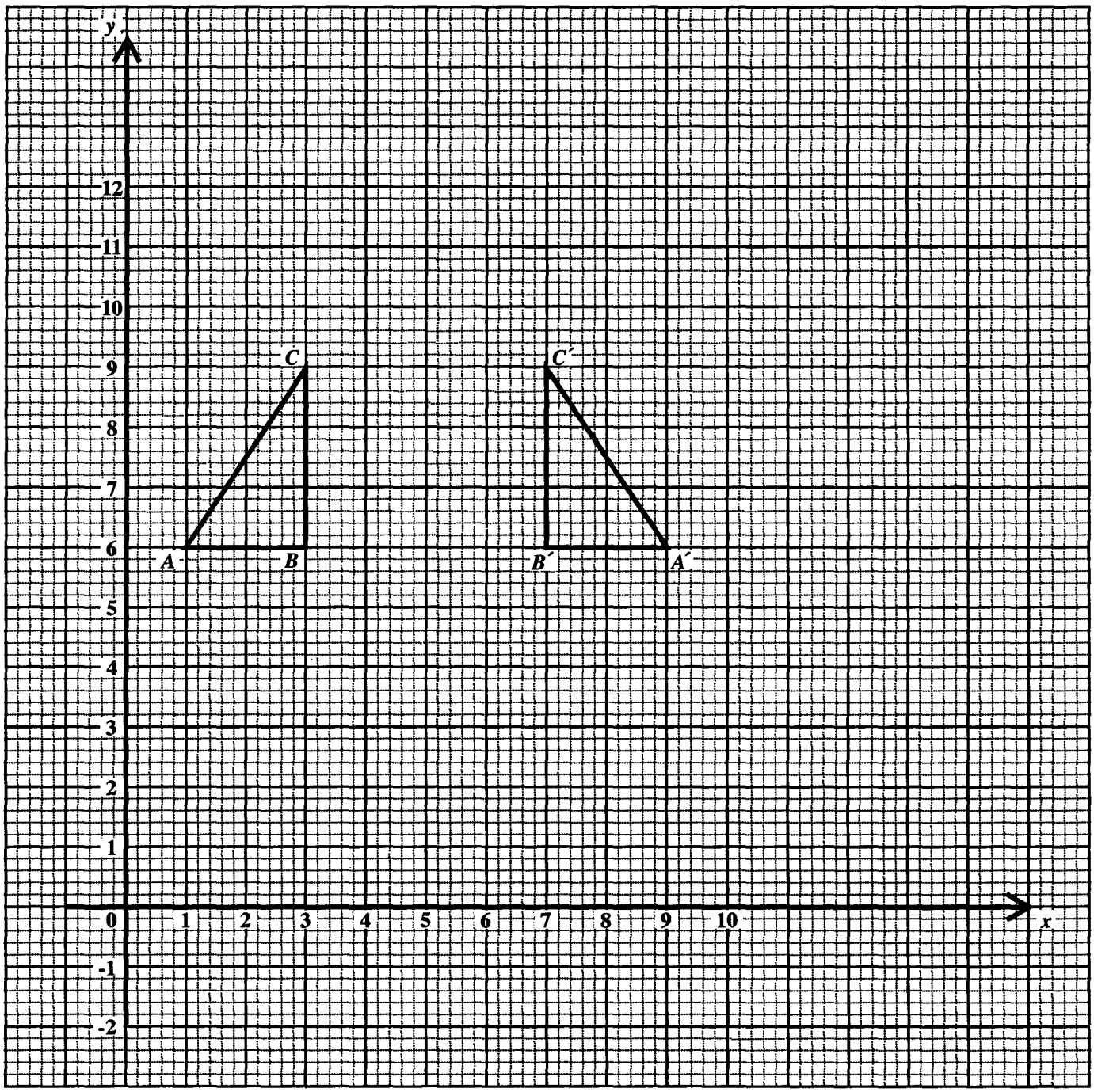
JANUARY 2007

MATHEMATICS

Paper 02 – General Proficiency

Answer Sheet for Question 5 (b) **Candidate Number .....**



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